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PRELIMINARY COMMUNICATION

Analytical theory for field induced periodic equilibrium structures in nematic and cholesteric films

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Planar films of cholesteric liquid crystals exhibit an instability in electric or magnetic fields. Depending on a special choice of the elastic constants and the helix pitch either a striped texture appears at a threshold field or an ordinary Freedericksz transition is observed. The type of field-induced transition can be predicted by a simple criterion.

Thin films of nematic and cholesteric liquid crystals in an electric field can have a periodic equilibrium state, which is visible as a striped texture [1, 2]. Chigrinov *et al.* [1] have determined the threshold of the stripe instability by numerical methods for cholesteric, planar oriented films with the helix axis parallel to the field. Numerical methods have also been used by Lonberg *et al.* [2] to investigate periodic deformations of a planar nematic layer in a field. Recently, Allender has claimed [3] that the phase diagram of nematic and cholesteric films contains a Liftshitz point at which the wavevector of the modulated phase goes to zero. In this preliminary communication we present some results of an analytical theory for Lifshitz points in liquid crystal layers subjected to an external field. It turns out to be possible to locate Lifshitz points in phase diagrams by a simple procedure.

In figure 1 the geometry of a planar cholesteric layer is shown. The director is fixed at the plate surfaces $X = 0$ and $X = d$. The angle θ between the director and the plane $X =$ constant, is zero below the threshold voltage of an instability. The azimuthal director rotation, Φ, increases gradually with X and has its maximum $\Phi = \alpha$ at the upper plate. The Z axis of the cartesian coordinate system is parallel to the stripes and the Y axis is parallel to the wavevector of a periodic distortion. This wavevector and the director at the lower plate enclose a definite angle κ. For convenience we use dimensionless coordinates

$$x = \frac{\pi X}{d} \quad \text{and} \quad y = \frac{\pi Y}{d}, \tag{1}$$

so that the film is located within the interval $0 \leqslant x \leqslant \pi$. Furthermore, the notation

$$\left.\begin{aligned} k_2 &= \frac{K_{22}}{K_{11}}, \quad k_3 = \frac{K_{33}}{K_{11}}, \quad \omega = \frac{\alpha}{\pi}, \quad \beta = \frac{2\pi d}{P\alpha}, \\ \Omega &= \omega x + \kappa \quad \text{and} \quad \gamma = \frac{\varepsilon_{\parallel} - \varepsilon_{\perp}}{\varepsilon_{\perp}} \ll 1, \end{aligned}\right\} \tag{2}$$

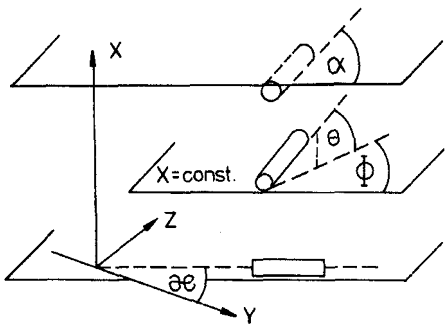

Figure 1. The geometry of a cholesteric film. θ is the angle between the director and the plane Z = constant; Φ is the azimuthal angle of the director; and α is the maximum twist angle. The Z axis of the cartesian coordinate system is parallel to the stripes and the angle κ is enclosed by the wavevector of the periodic distortion and the director at the lower plate.

is introduced. K_{11}, K_{22} and K_{33} denote the elastic constants, which are defined in the framework of the Oseen–Frank theory [4], P is the helix pitch of a cholesteric liquid crystal, $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ are the dielectric constants measured parallel and perpendicular to the director, respectively ($\varepsilon_{\parallel} > \varepsilon_{\perp}$). The director configuration can be calculated by a perturbation theory [5], which is valid for small distortions. In lowest order of magnitude

$$\left.\begin{aligned} \theta &= b(y)\sin x \\ \Phi &= \Omega + \varphi(x) b_y \end{aligned}\right\} \qquad (3)$$

and

are obtained close to the Lifshitz point [6], where $\varphi(x)$ is given later and $b_y = (db/dy)$. The results for non-twisted nematic layers ($\alpha = \beta = 0$) and cholesteric layers will be discussed separately.

For $\omega = 0$ the function $\gamma(x)$ in equation (3) is obtained as

$$\varphi(x) = \left(\frac{1-k_2}{k_2}\right)\left(1 - \frac{2}{\pi}x - \cos x\right), \qquad (4)$$

where Ω is equal to $\pi/2$ and the distortion amplitude b satisfies the differential equation

$$-Hb_{yyyy} + Db_{yy} + \mu b - Bb^3 = 0, \qquad (5)$$

where

$$\left.\begin{aligned} H &= \frac{(1-K)^2}{\pi^2}\left(3 - \frac{16-\pi^2}{\pi^2-8} + \frac{5\pi^2-48}{6K^2}\right), \\ B &= \frac{k_3+\gamma}{4}, \\ \mu &= \frac{U-U_F}{U_F} \\ U_F &= \sqrt{\left(\frac{\pi^2 K_{11}}{\varepsilon_0(\varepsilon_{\parallel}-\varepsilon_{\perp})}\right)}. \\ D &= -\frac{(1-k_2)^2}{2k_2}\left(1-\frac{8}{\pi^2}\right) + \frac{k_2}{2}, \end{aligned}\right\} \qquad (6)$$

with

or, by neglecting terms proportional to $(k_2 - K)^2$, we find

$$D = \frac{1}{2}\left[\frac{1-K^2}{K^2}\left(1 - \frac{8}{\pi^2}\right) + 1\right](k_2 - K).$$

(Here ε_0 is the dielectric constant of the vacuum and U is the applied voltage.) The parameter

$$K = \frac{\sqrt{[1 - (8/\pi^2)]}}{1 + \sqrt{[1 - (8/\pi^2)]}} \approx 0{\cdot}303 \tag{7}$$

has already been found numerically in [2] and analytically in [7]. Equation (5) is well known in the general theory of Lifshitz points, which also occur in thermodynamic systems. Using the coefficients of equation (5) a phase diagram can be constructed [8]. The location of the Lifshitz point is determined by

$$\mu = 0 \quad \text{and} \quad D = 0, \tag{8}$$

and the corresponding phase diagram is shown in figure 2. For $k_2 < K$ first the stripe instability occurs with increasing voltage by a continuous transition (that is a second order phase transition). In the other case, $k_2 > K$, the ordinary Freedericksz transition takes place immediately. The topological features of the phase diagram in figure 2 remain valid for Lifshitz points of cholesteric films.

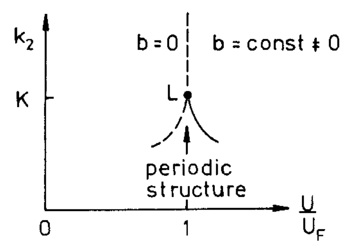

Figure 2. Phase diagram of a nematic layer ($\alpha = \beta = 0$). k_2 is the ratio of the elastic constants K_{22} and K_{11}; L is the Lifshitz point; U is the applied voltage; and U_F is the Freedericksz threshold. ——, discontinuous transition; - - -, continuous transition.

For cholesteric planar films $\varphi(x)$ is found from equation (3) to be [6]

$$\varphi(x) = -\frac{\pi - x}{\pi}\int_0^x \xi f(\xi)d\xi - \frac{x}{\pi}\int_x^\pi (\pi - \xi)f(\xi)\,d\xi, \tag{9}$$

where the function $f(x)$ is defined by

$$f(x) = \left(\frac{1-k_2}{k_2}\right)\sin\Omega\cos x - \frac{(k_3 - k_2 + 2k_2\beta}{k_2}\,\omega\cos\Omega\sin x. \tag{10}$$

Now the coefficient D in equation (5) depends on the angle κ (see figure 1)

$$D(\kappa) = \frac{1}{\pi}\int_0^\pi dx\sin x\left[-\frac{d\varphi(x)}{dx}(1 - k_2)\sin\Omega \right.$$
$$- \varphi(x)\,(1 - 2k_2 + k_3 + 2k_2\beta)\omega\cos\Omega$$
$$\left. + (k_2\sin^2\Omega + k_3\cos^2\Omega)\sin x\right]. \tag{11}$$

For a Lifshitz point the condition

$$\min(D(\kappa)) = 0 \tag{12}$$

is satisfied and the corresponding voltage is

$$U_F = \sqrt{\left\{\frac{\pi^2 K_{11}}{\varepsilon_0(\varepsilon_\parallel - \varepsilon_\perp)}[1 + \omega^2(k_3 - 2k_2 + 2k_2\beta)]\right\}}. \tag{13}$$

If $\min(D(\kappa)) < 0$ the striped texture emerges from the initial state $b = 0$ at a voltage $U_S < U_F$. In the other case, namely if $\min(D(\kappa)) > 0$, the ordinary Freedericksz transition appears at the threshold voltage (13).

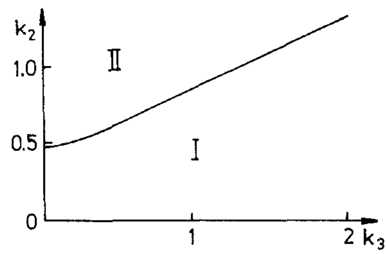

Figure 3. Regions of the stripe instability (I) and the ordinary Freedercksz transition (II) for $\beta = 0{\cdot}5$ and $\alpha = \pi$. k_2 is the ratio of the elastic constants K_{22} and K_{11}; and k_3 is the ratio of the elastic constants K_{33} and K_{11}.

Clearly, if $\alpha = \beta = 0$ equation (12) leads to $\kappa = \pi/2$ and equation (7) for the critical value K of k_2 is derived. As a second example let us assume that $\alpha = \pi(\omega = 1)$. Then equation (12) results in $\kappa = \pi/2$ and Lifshitz points satisfy the condition

$$4k_2^2 + 12k_2k_3 + 8(1 - k_2)^2 - 3(k_3 - 1 + 2k_2\beta)^2$$
$$- 2\left(\frac{\pi^2 + 3}{3}\right)(k_3 + 1 - 2k_2 + 2k_2\beta)^2 = 0; \tag{14}$$

k_3 is plotted as a function of k_2 in figure 3 for $\beta = 0{\cdot}5$. The set of Lifshitz points separates the phase diagram into the region of the stripe instability and the region of the ordinary Freedericksz transition.

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